8th INTERNATIONAL SCIENTIFIC CONFERENCE
ON DEFENSIVE TECHNOLOGIES
OTEH 2018
Belgrade, Serbia, 11 – 12 October 2018

CONNECTION OF THE TIME AND LEVEL OF CROSS-CORRELATION OF SIGNALS OF A COMPLEX TARGET WITH MAGNITUDE OF SPACING OF THE ANALYZERS

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Abstract: *The article treats the problem of connection between the time of correlations of fluctuations and the radius of spatial correlation of complex amplitudes of signals of a complex target. Models of a "rigid" and "non-rigid" targets are viewed separately. It is asserted that signals received by a pair of analyzers spaced apart by a distance exceeding the radius of spatial correlation of the signals retain strong cross-correlation.*

Keywords: *spatial correlation radis, fluctuations correlation time, spaced-apart receivers-analyzers.*

1. INTRODUCTION

To perform operations of synthesis and analysis of multidimensional adaptive processors of coherent multiposition radars, one has to have a model of the received signal from a complex multi-element target. The article discusses a mathematical model of the received signal, its correlation properties. The conditions of separability of time-space processing and quasi-monochromatic approximation are presumed to be met (radial dimensions of the target do not exceed the range resolution of the signal). The issue of connection between the time of correlations of fluctuations and the radius of spatial correlation of complex amplitudes of signals of a complex target is considered. Models of a "rigid" and a "non-rigid" targets are treated separately. It is asserted that signals received by a pair of analyzers spaced apart by a distance exceeding the radius of spatial correlation of the signals retain strong cross-correlation. Possibility is discussed of space-coherent processing across the mouths with the dimensions exceeding the radius of spatial coherence of the received fields.

2. MATHEMATICAL MODEL OF THE RECEIVED SIGNAL

A complex multi-element target (CT) representing a totality of illuminated objects re-reflects the illuminating field in the direction of the system of receivers-analyzers (RA). Based on extensive research [1 - 5], a complete field reflected from a CT can be presented in the form of the sum of reflections from separate local reflectors (LR). Accordingly, the expression for the signal at the output of the RA, from which the elements of the CT are seen at the angle γ to normal to the aperture, can be written in the form:

$$m(t,\gamma)=\sum_{j=0}^{M-1}A_j(t,\gamma)\cdot \exp\left\{i\left[(\omega_0+\Omega_{d0})t-k\vec{l}_{0j}\vec{x}_0\sin\gamma+\xi(t,\gamma)\right]\right\}, \quad (1)$$

where $A_j(t,\gamma)=a_j(t,\gamma)\exp\{i\cdot\phi_j(t,\gamma)\}$ - is the complex secondary backscatter pattern (SBSP) of the j-th aerial vehicle (AV); ω_0 - is carrier frequency of the probing signal; $\vec{l}_{0j}$ - is the radius-vector connecting the phase centers of the 0-th and the j-th AVs; $\xi(t,\gamma)$ - is the random phase incursion along the track, conditioned by irregularities of the refraction indicator; Ω_{d0} - is the Doppler shift of the 0-th AV circular frequency. The differences in Doppler shifts of secondary radiation of AVs $\Delta\Omega_{dj}$ can be taken into account using the expression $\phi_j(t,\gamma)=\Phi_j(t,\gamma)+\Delta\Omega_{dj}t$, where $\Phi_j(t,\gamma)$ - is phase SBSP of the j-th AV.

3. CONNECTION OF THE TIME AND LEVEL OF CROSS-CORRELATION AND SPACING OF THE ANALYZERS

3.1. Model of a "rigid" target

Let us determine the cross-correlation function of signals received simultaneously at the angles γ_1 and γ_2: $R(\gamma_1,\gamma_2)=M\left\{\overline{m(t,\gamma_1)\cdot m^*(t,\gamma_2)}\right\}$, where the operator $M\{\cdot\}$ denotes averaging by the ensemble, and the straight line – time averaging. Taking into account [3, 6] and the condition of quasi-monochromaticity and limiting ourselves by the case of a small observation sector, one can write for the case of the "rigid" target:

$$R(\gamma_1,\gamma_2)=\sum_{m,n=0}^{M-1}\{2\sigma_m(\gamma_1)\sigma_n(\gamma_2)r_{mn}(0)e^{i\phi_{mn}}\times \times M\{A_m(\gamma_1)A_n^*(\gamma_2)\}\cdot g(V_1,V_2)\}, \quad (2)$$

where $2\sigma_j^2(\gamma_k)$ - is averaged power of the signal of the j-th AV received from the angular direction γ_k; r_{mn}, ϕ_{mn} - is the cross-correlation function and average difference of the initial phases of signals of the m-th and *n*-th AVs; $g(v_1,v_2)=M\{\exp\{i[v_1\gamma_1 - v_2\gamma_2]\}\}$ - is the response function of two-dimensional distribution of the weighted sum of angles γ_1 and γ_2; $v_1=-kd_{0m}$, $v_2=-kd_{0n}$; $d_{0k}=\vec{l}_{0k}\vec{x}_0$.

The obtained expression for the cross-correlation function of complex amplitudes of signals is analogous to the expression for the time correlation function of radar cross section (RCS) of a complex target [1]. This justifies interpreting the cause of randomness of a signal re-reflected by a "rigid" CT as the result of its angular yawing. In addition, one can describe interrelation of the time of autocorrelation of fluctuations of the CT signal complex amplitudes τ_s and its spatial correlation radius δl_s in the form of the correlation [1]: $\tau_s=Q\frac{\lambda}{L_s}\times\frac{1}{\sigma_\gamma}=Q'\frac{\lambda}{L_s}=Q'\cdot dl_s$, where Q and Q′ - are proportionality coefficients; λ/L_s - is the maximum electrical dimension of the complex target; σ_γ - is the root mean square value of the target yaw rate. The meaning of the latter expression consists in that the maximum spatial spacing of a RA whereby spatial correlation of the registered signals is retained and is directly proportional to the time of autocorrelation of the received signal fluctuations. Note that the above interconnection does not mean that, in case the pair of analyzers is spaced apart by a distance exceeding dl_s, cross correlation of the received signals will disappear simultaneously with disappearance of the spatial correlation.

As an example, we will view a "rigid" target representing two spheres with equal RCS, connected by a radio transparent rod of l_0 length. Calculations show that, with accuracy down to an inexistent factor, the function $R(\gamma_1,\gamma_2)=\cos(kl_0\cos\gamma_1)\cos(kl_0\cos\gamma_2)$ is periodic. The role of the pattern of a pair of reflectors consists in that, at angles γ_1 and γ_2 corresponding to zero reception, the correlation function zeroes out. However, this does not mean disappearance of statistical interrelations of the received signals, since the normalized correlation factor $r(\gamma_1,\gamma_2)=\frac{|R(\gamma_1,\gamma_2)|}{\sqrt{R(\gamma_1,\gamma_1)R(\gamma_2,\gamma_2)}}$, for the assumptions adopted, is equal to one for any γ_1 and γ_2, and is limited only by the influence of atmospheric irregularities.

If a time mismatch $R(\tau,\gamma_1,\gamma_2)=\overline{m(t,\gamma_1)m^*(t-\tau,\gamma_2)}$ is introduced, it is possible to determine integral time of cross-correlation of the received signals $\tau_s(\gamma_1,\gamma_2)=0.5\int_{-\infty}^{\infty}r(\tau,\gamma_1,\gamma_2)d\tau$, infinite in "frozen" atmosphere and, taking into account the atmosphere, equal to $\tau_s(\gamma_1,\gamma_2)=\frac{\tau_a}{2\sigma_\phi(\gamma_1)\sigma_\phi(\gamma_2)}\approx\frac{\tau_a}{2\sigma_\phi^2}$, where σ_ϕ^2 - is dispersion of phase distortions of the field; τ_a - is the time of correlation of fluctuations of atmospheric irregularities. As one can see, in the case examined, the coefficient and time of cross-correlation of signals received by the spaced-apart signal analyzers do not decrease with the spacing $\gamma_1-\gamma_2$ grown, moreover, they do not depend on it at all.

3.2. Model of a "non-rigid" target

In a "non-rigid" target model, signals re-reflected by LRs are arbitrarily cross-correlated. Each LR has finite dimensions and a secondary backscatter pattern limited in width. Randomness of a signal reflected from a CT is caused by fluctuations of the signal-summands from separate LRs which, in their turn, are caused by their arbitrarily correlated angular yawing. The expression for the time cross-correlation function of CT signals received at different γ_1 and γ_2 angles taking into account [6] will be written as:

$$R_t(t_1,t_2,\gamma_1,\gamma_2)=\sum_{m,n=0}^{M-1}\overline{\{A_m(t_1,\gamma_1)A_n^*(t_2,\gamma_2)}\times \times\exp\{-\sigma_\phi^2[1-R_\xi^{mn}(t_1-t_2,\gamma_1,\gamma_2)]\}\times \times\exp\{-ik[d_{om}\sin\gamma_1-d_{0n}\sin\gamma_2]\}\times \times\exp\{i[(\omega_0+\Omega_{d0})(t_1-t_2)]\}\}, \quad (3)$$

where $R_\xi^{mn}(t_1-t_2,\gamma_1,\gamma_2)=M\{\xi_m(t_1,\gamma_1)\xi_n(t_2,\gamma_2)/\sigma_\phi^2\}$.

For more detailed analysis of the correlation (3), we will examine an example. Reception of signals of non-cross-correlated LRs takes place at coinciding moments of time ($t_1=t_2$). Assuming that $R_\xi^{mn}(0,\gamma_1,\gamma_2)\to 0$ and for the case of minor angular spacing of the analyzers, one can obtain:

$$R_t(\gamma_1,\gamma_2)=\sum_{m=0}^{M-1}\{2\sigma_m(\gamma_1)\sigma_m(\gamma_2)\times \times\exp\{-ikd_{om}[\sin\gamma_1-\sin\gamma_2]\}\}. \quad (4)$$

In accordance with (4), one will notice that spacing of the analyzers by an angular distance exceeding the width of the secondary backscatter pattern lobe dl_s, does not result in disappearance of cross-correlation of signals they receive. Separate points stand as an exception, where zeroing out of $R_t(\gamma_1,\gamma_2)$ is connected with location of one (or both) analyzers in the function's null (4).

4. ANALYTICAL AND SIMULATION MODELING

In accordance with the expressions obtained, a series of analytical calculations was performed. The model of a fighter shown in Figure 1 was used for the purpose. 13 LRs are simulated, each of which possesses a rectangular AP, and "shading" of the LR is taken into account. The model simulates signals of both a "rigid" target and a "non-rigid" target (signals of separate LRs have arbitrary cross-correlation). The analytical model used enables good approximation in simulating SBSP of a CT radiation [7] and is useful for studying quality interrelations of average SBSP of a CT with the function $R(\tau,\gamma_1,\gamma_2)$.

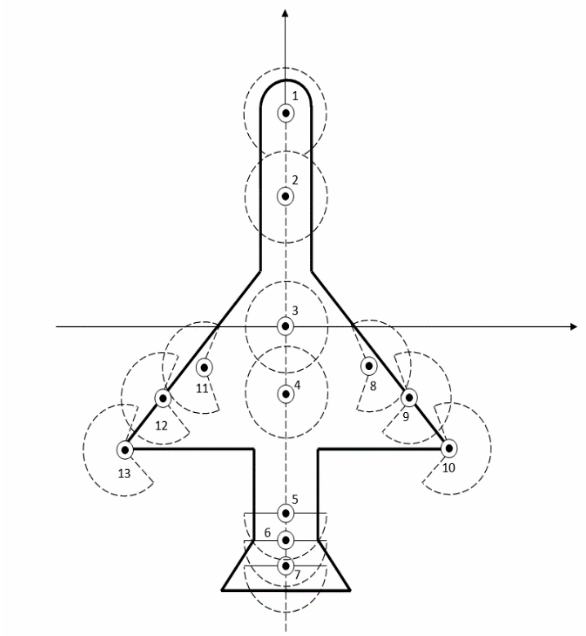

Figure 1. Multi-point fighter model

Calculations $R(0,\gamma_1,\gamma_2)$ were carried out in accordance with (3). Also, the normalized coefficient $r(\gamma_1,\gamma_2)$ and time $\tau(\gamma_1,\gamma_2)$ of cross-correlation of CT signals were calculated. For a "rigid" target, the results obtained demonstrated interrelation between the shape of SBSP and $R(0,\gamma_1,\gamma_2)$ and weak dependence of $r(0,\gamma_1,\gamma_2)$ on values of the observation angles for minor dispersion of angular yawing. It was also confirmed that the radius of spatial correlation of a CT signal is usually much lesser than the interval of strong cross-correlation of the received signals. For a "non-rigid" one, a considerable excess is observed of the interval of effective auto-coherent integration over the spread of the received signal spatial correlation radius. Analytical calculations show weak influence of angular spacing of a RA on the value of τ_s for a "rigid" target and a drastic shrinking of the time of cross-correlation of signals of a RA with weakening of cross-correlation of LR signals. Thus, one can conclude from analytical modeling that using a model of "local reflectors" of a CT demonstrates weak dependence of the time of cross-correlation of the received signals on angular misregistration of the RA as well as presence of strong cross-correlation of CT signals in case of misregistration of a RA considerably exceeding the mean width of their secondary backscatter pattern lobe.

For experimental confirmation of the conclusions made, simulation modeling of complex amplitudes of the received signals of two RAs was carried out, using the special software package, BSE Electrodynamic modeling of a radar signal reflected by a moving object of complex shape" [8]. Yawing of targets in heading, roll and pitch was simulated by "pushes" connected with random irregularities of air pressure. The experiments resulted in recordings of samples of the received signal complex amplitudes, as well as cross-correlation functions of signals components of the received signal calculated in accordance with (1) and (2), normalized to maximum values. In particular, figure 2 shows the results of calculations of the dependence of the cross-correlation function of signals R on time, performed for a target of MiG-23 type (target altitude $H_t = 1\text{ km}$, target range $r_t = 11\text{ km}$, spacing of the PA 20 m, wavelength $\lambda = 4\text{ cm}$). For figure 3 a: tangential speed component $V_\tau = 200\text{ m/s}$, radial speed component $V_r = 0\text{ m/s}$; 3 б: $V_\tau = 0\text{ m/s}$, $V_r = 200\text{ m/s}$. Analysis of the data obtained shows that, during radial flight of a target, cross-correlation time is rather protracted, exceeding tens of milliseconds in practical situation, whereas the cross-correlation ratio is close to one. Of most interest is the situation of tangential target flight, since alteration of the SBSP lobes takes place with maximum speed. The time of cross-correlation of signals of a fighter in this case exceeds 10 mc.

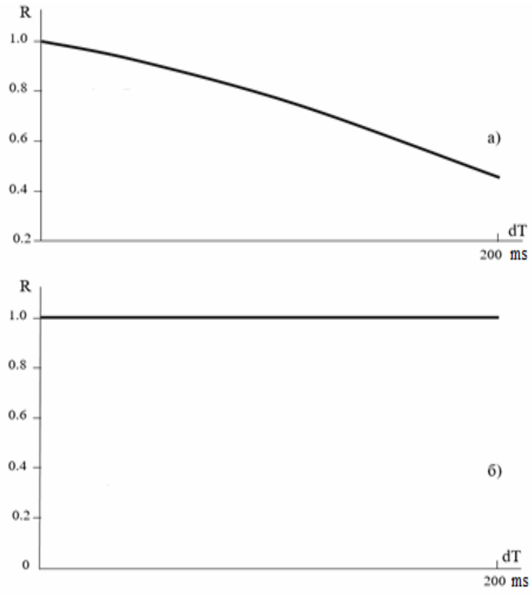

Figure 2. Dependence of cross-correlation function of signals on time

5. CONCLUSION

1. Unambiguous linear correlation between the time and radius of spatial correlation of fluctuations of a signal re-reflected by a CT is typical only for targets with rigid structural interrelation of elements, whereas for targets whose local reflectors possess relative freedom

of translation, there is no such unambiguity.

2. Signals received by a pair of analyzers spaced apart by a ΔX distance exceeding the radius of spatial correlation of signals of a complex target dl_s, retain strong cross-correlation even at $\Delta X >> \delta l_s$, whereby $|R_t(\Delta x)|$ grows in proportion to increase of the width of the secondary backscatter pattern radiated by separate local reflectors.
3. A drop of the cross-correlation factor of a CT signals received by spaced-apart RAs takes place with a lteration of the AVs "visible" at corresponding angles, and the weaker is cross-correlation of the fields they re-reflect, the faster this process is.

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